Cutting Up and Taping Together

James King
University of Washington
king@math.washington.edu
www.math.washington.edu/~king

Scaling up a triangle

Cut out the triangles on your first sheet.

- Place some of the triangles together to form a new triangle, similar to the original, but with all sides twice a large as the original. How many triangles does this take?
- Now build a triangle with sides 3 times as long. How many triangles?
- Next, build a triangle with sides 4 times as long. How many triangles?
- What will happen if you build a triangle n times as long?
- How is this related to an important fact about area and scaling?

Notice that in your triangles above, some of the small congruent triangles are pointing "up" and some "down". In each set of triangles, each of the triangles is a translation of any of the others.

Count the number of "up" triangles and the number of "down" triangles. For each of the sizes above, side length = 1, 2, 3, 4, … times the original.

Make a table and look for a pattern.

N	#up	#down
1		
2		
3		
4		

What relationships, formulas, etc. can you find here?

Merging Two Triangles to Scale Up

Cut out the triangles from you second sheet.

Now you have a set of Acute isosceles triangles that we will call A's and Obtuse isosceles triangles that we will call O's.

- Put one A and one O together to form a new triangle A1. Can you convince yourself that this new triangle is similar to A?

- If in fact the triangle A1 is similar to A, use the relations that you see to deduce the **measure of each of the angles** of the triangles A and O.

Counting

Now combine some A's and O's to form a triangle O1 that is similar to O and is scaled up by the same scaling ratio as A to A1. In other words, A1 and O1 should fit together just as A and O do. How many A's and how many O's does it take to make O1.

Now we will continue building up bigger A's and O's by exactly the same method. Form an A2 from A1 and O1 as A1 was formed from A and O. And the same for O2.

Now again, we will make a table of the number of A's and O's to make each shape. Look for patterns. Can you figure out how to compute the numbers without making the shape? What is the rule?

N	#As	#Os	
A			
A1			
A2			
A3			
A4			
A5			

N	#As	#Os	
O			
O1			
O2			
O3			
O4			
O5			

In the last column, write the ratio of O/A in each row (if you have a calculator). Does this ratio appear to approach a limit?

The Ratios of the Sides in the Triangles

- Study the figure of A1, which is made of one A and one O. You have already found the angles from this figure. Use the fact that A and A1 are similar and that all 3 triangles are isosceles to find the ratio of the long side to the short side of A.

Regular Polygon

Form a regular polygon P from the smallest number of A's and O's. What is the shape of this Polygon?

Then form larger polygons P1, P2, etc. from the larger A1, A2, etc. Again, make a table of the number of A's and O't that make up Pn. What is the pattern here?

N	#As	#Os	
P			
P1			
P2			
P3			
P4			
P5			

Silver Rectangles and Similarity

Take a regular sheet of 8.5x11 inch paper and fold the long sides in half to form two smaller "half" rectangles.

Is the half rectangle similar to the original? How can you tell visually? How can you tell numerically?

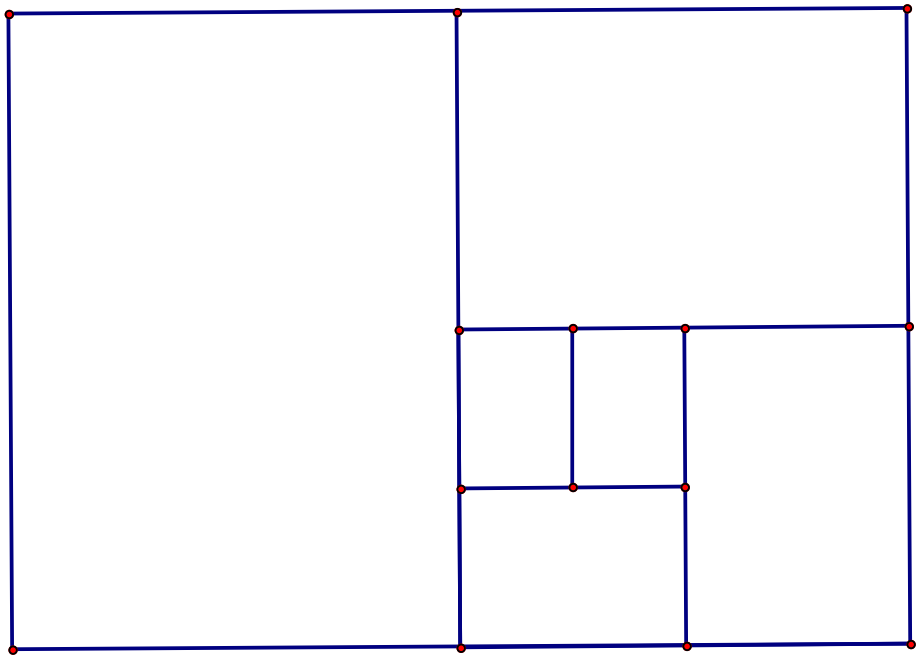
Now take a new cut-down sheet of paper and fold it in the same way into half rectangles.

- Is the half rectangle similar to the original? How can you tell visually**? How can you tell numerically?**
- A rectangle with this property that half the rectangle is similar to the whole is called a silver rectangle. If you want to cut down a regular sheet of paper to a silver sheet, so that the length is still 11 inches, what is the width?

Silver Spirals

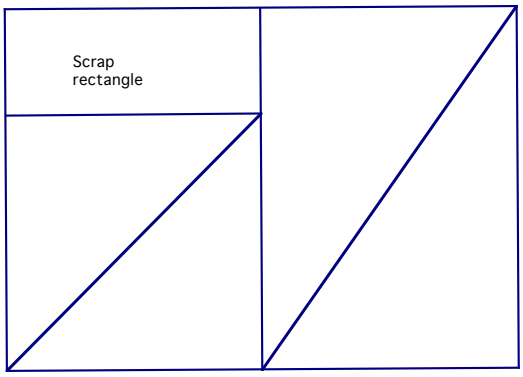
Take a silver rectangle R and cut it in half to make two smaller rectangles, then cut one of the smaller ones in half and continue as shown

The rectangles spiral in to a limit point. Can you locate this point by geometry or numerically?

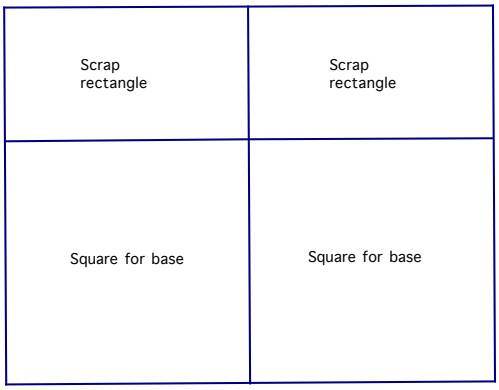


A Silver Pyramid

Cut up a silver rectangle into 4 right triangles (plus a scrap rectangle) as shown in this diagram (using folding to produce the cut lines).



Also, cut one extra square from an 8.5 x 11 sheet as shown.



Now with the square as the base, tape together a square-based pyramid.

Then take 3 of the pyramids and form a regular polyhedron.

What are the edge lengths of these shapes? What is the volume of the pyramid as compared to the volume of the polyhedron.